

Evolutionary Games for Multiple Access Control^{*†}

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January 14, 2013

Abstract

In this paper, we formulate an evolutionary multiple access control game with continuous-variable actions and coupled constraints. We characterize equilibria of the game and show that the pure equilibria are Pareto optimal and also resilient to deviations by coalitions of any size, i.e., they are strong equilibria. We use the concepts of price of anarchy and strong price of anarchy to study the performance of the system. The paper also addresses how to select one specific equilibrium solution using the concepts of normalized equilibrium and evolutionarily stable strategies. We examine the long-run behavior of these strategies under several classes of evolutionary game dynamics, such as Brown-von Neumann-Nash dynamics, Smith dynamics and replicator dynamics. In addition, we examine correlated equilibrium for the single-receiver model. Correlated strategies are based on signaling structures before making decisions on rates. We then focus on evolutionary games for hybrid additive white Gaussian noise multiple access channel with multiple users and multiple receivers, where each user chooses a rate and splits it over the receivers. Users have coupled constraints determined by the capacity regions. Building upon the static game, we formulate a system of hybrid evolutionary game dynamics using G-function dynamics and Smith dynamics on rate control and channel selection, respectively. We show that the evolving game has an equilibrium and illustrate these dynamics with numerical examples.

1 Introduction

Recently, there has been much interest in understanding the behavior of multiple access controls under constraints. Considerable amount of work has been carried out on the problem of how users can obtain an acceptable throughput by choosing rates independently. Motivated by the interest in studying a large population of users playing the game over time, evolutionary game theory was found to be an appropriate framework for communication networks. It has been applied to problems such as power control in wireless networks and mobile interference control [11, 1, 5, 6].

The game-theoretical models considered in the previous studies on user behaviors in CDMA, [37, 4], are static one-shot non-cooperative games in which users are assumed to be rational and optimize their payoffs independently. Evolutionary game theory, on the other hand, studies games that are played repeatedly, and focuses on the strategies that persist over time, yielding the best fitness of a user in a non-cooperative environment on a large time scale.

In [19], an additive white Gaussian noise (AWGN) multiple access channel problem was modeled as a noncooperative game with pairwise interactions, in which users were modeled as rational

^{*}The material in this paper was partially presented in [9] and [10].

[†]This work was supported in part by a grant from AFOSR and by a MURI grant.

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entities whose only interest was to maximize their own communication rates. The authors obtained the Nash equilibrium of the two-user game and introduced a two-player evolutionary game model with *pairwise interactions* based on replicator dynamics. However, the case when interactions are not pairwise arises frequently in communication networks, such as the Code Division Multiple Access (CDMA) or the Orthogonal Frequency-Division Multiple Access (OFDMA) in Worldwide Interoperability for Microwave Access (WiMAX) environment [11].

In this work, we extend the study of [19] to wireless communication systems with an arbitrary number of users corresponding to each receiver. We formulate a static non-cooperative game with m users subject to rate capacity constraints, and extend the constrained game to a dynamic evolutionary game with a large number of users whose strategies evolve over time. Different from evolutionary games with discrete and finite number of actions, our model is based on a class of continuous games, known as *continuous-trait games*. Evolutionary games with continuum action spaces can be encountered in a wide variety of applications in evolutionary ecology, such as evolution of phenology, germination, nutrient foraging in plants, and predator-prey foraging [24, 7].

In addition to the single receiver model, we investigate the case with multiple users and receivers. We first formulate a static hybrid non-cooperative game with N users who rationally make decisions on the rates as well as the channel selection subject to rate capacity constraints of each receiver. We extend the static game to a dynamic evolutionary game by viewing rate selections governed by a fitness function parameterized by the channel selections. Such a concept of a hybrid model has appeared earlier in [36] and [40], in the context of hybrid power control in CDMA systems. The strategies of channel selections determine the long-term fitness of the rates chosen by each user. We formulate such dynamics based on generalized Smith dynamics and generating fitness function (G-function) dynamics.

1.1 Contribution

The main contributions of this work can be summarized as follows. We first introduce a game-theoretic framework for local interactions between many users and a single receiver. We show that the static continuous-kernel rate allocation game with coupled rate constraints has a convex set of pure Nash equilibria, coinciding with the maximal face of the polyhedral capacity region. All the pure equilibria are Pareto optimal and are also strong equilibria, resilient to simultaneous deviation by coalition of any size. We show that the pure Nash equilibria in the rate allocation problem are 100% efficient in terms of Price of Anarchy (PoA) and constrained Strong Price of Anarchy (CSPoA). We study the stability of strong equilibria, normalized equilibria, and evolutionary stable strategies (ESS) using evolutionary game dynamics such as Brown-von Neumann-Nash dynamics, generalized Smith dynamics, and replicator dynamics. We further investigate the correlated equilibrium of the multiple access game where the receiver can send signals to the users to mediate the behaviors of the transmitters.

Based on the single-receiver model, we then propose an evolutionary game-theoretic framework for the hybrid additive white Gaussian noise multiple access channel. We consider a communication system of multiple users and multiple receivers, where each user chooses a rate and splits it over the receivers. Users have coupled constraints determined by the capacity regions. We characterize Nash equilibrium of the static game and show the existence of the equilibrium under general conditions. Building upon the static game, we formulate a system of hybrid evolutionary game dynamics using G-function dynamics and Smith dynamics on rate control and channel selection, respectively. We show that the evolving game has an equilibrium and illustrate these dynamics with numerical examples.

Table 1: List of Notations

Symbol	Meaning
\mathcal{N}	set of N users
Ω	a subset of N users
\mathcal{J}	set of J receivers
\mathcal{A}_i	action set of user i
P_i	maximum power of user i
h_i	channel gain of user i
α_i	rate of user i
p_{ij}	probability of user i selecting receiver j
u_i	payoff of user i
\bar{U}_i	expected payoff of user i
\mathcal{C}	capacity region of a set \mathcal{N} of users in a single receiver case
$\mathcal{C}(j)$	capacity region of a set \mathcal{N} of users at receiver j
λ_i	distribution over the action set \mathcal{A}_i
μ	population state

1.2 Organization of the paper

The rest of the paper is structured as follows. We present in Section 2.1 the evolutionary game model of rate allocation in additive white Gaussian multiple access wireless networks, and analyze its equilibria and Pareto optimality in Section 2.2. In Section 2.3, we present strong equilibria and price of anarchy of the game. In Section 2.4, we discuss how to select one specific equilibrium such as normalized equilibrium and evolutionary stable strategies. Section 2.5 studies the stability of equilibria and evolution of strategies using game dynamics. Section 2.6 analyzes the correlated equilibrium of the multiple access game.

In Section 3.1, we present the hybrid rate control model where users can choose the rates and the probability of the channels to use. In Section 3.2, we characterize the Nash equilibrium of the constrained hybrid rate control game model, pointing out the existence of the Nash equilibrium of the hybrid model and methods to find it. In Section 3.3, we apply evolutionary dynamics to both rates and channel selection probabilities. We use simulations to demonstrate the validity of these proposed dynamics and illustrate the evolution of the overall evolutionary dynamics of the hybrid model. Section 4 concludes the paper. For reader's convenience, we summarize the notations in Table 1 and the acronyms in Table 2.

2 AWGN Mutiple Access Model: Single Receiver Case

We consider a communication system consisting of several receivers and several senders (see Figure 1). At each time, there are several simultaneous local interactions (typically, at each receiver there is a local interaction). Each local interaction corresponds to a non-cooperative one-shot game with common constraints. The opponents do not necessarily stay the same from a given time slot to the next one. Users revise their rates in view of their payoffs and the coupled constraints (for example by using an evolutionary process, a learning process or a trial-and-error updating process). The game evolves over time. Users are interested in maximizing a fitness function based on their own

Table 2: List of Acronyms

Abbreviation	Meaning
AGWN	Additive White Gaussian Noise
MAC	Multiple Access Control
MISO	Multi-Input and Multi-Output
CCE	Constrained Correlated Equilibrium
ESS	Evolutionary Stable Equilibrium
NE	Nash Equilibrium
PoA	Price of Anarchy
SPoA	Strong Price of Anarchy

communication rates at each time, and they are aware of the fact that the other users have the same goal. The coupled power and rate constraints are also common knowledge. Users have to choose independently their own coding rates at the beginning of the communication, where the rates selected by a user may be either deterministic, or chosen from some distribution. If the rate profile arrived at as a result of these independent decisions lies in the capacity region, users will communicate at that operating point. Otherwise, either the receiver is unable to decode any signal and the observed rates are zero, or only one of the signals can be decoded. The latter occurs when all the other users are transmitting at or below a safe rate. With these assumptions, we can define a constrained non-cooperative game. The set of allowed strategies for user i is the set of all probability distributions over $[0, +\infty[$, and the payoff is a function of the rates. In addition, the rational action (rate) sets are restricted to lie in the capacity regions (the payoff is zero if the constraint is violated). In order to study the interactions between the selfish or partially cooperative users and their stationary rates in the long run, we propose to model the problem of rate allocation in Gaussian multiple access channels as an evolutionary game with a continuous action space and coupled constraints. The development of evolutionary game theory is a major contribution of biology to competitive decision making and the evolution of cooperation. The key concepts of evolutionary game theory are (i) *Evolutionary Stable States* [27], which is a refinement of equilibria, and (ii) *Evolutionary Game Dynamics* such as replicator dynamics [32], which describes the evolution of strategies or frequencies of use of strategies in time, [7, 21].

The single population evolutionary rate allocation game is described as follows: there is one population of senders (users) and several receivers. The number of senders is large. At each time, there are many one-shot games called *local interactions*. Each sender of the population chooses from his set of strategies \mathcal{A}_i which is a non-empty, convex and compact subset of \mathbb{R} . Without loss of generality, we can suppose that user i chooses its rate in the interval $\mathcal{A}_i = [0, C_{\{i\}}]$, where $C_{\{i\}}$ is the rate upper bound for user i (to be made precise shortly), as outside of the capacity region the payoff (as to be defined later) will be zero. Let $\Delta(\mathcal{A}_i)$ be the set of probability distributions over the pure strategy set \mathcal{A}_i . The set $\Delta(\mathcal{A}_i)$ can be interpreted as the set of mixed strategies. It is also interpreted as the set of distributions of strategies among the population. Let $\lambda_i \in \Delta(\mathcal{A}_i)$, and E be a λ_i -measurable subset of \mathbb{R}^N ; then $\lambda_i(E)$ represents the fraction of users choosing a strategy out of E , at time t . A distribution $\lambda_i \in \Delta(\mathcal{A}_i)$ is sometimes called the “state” of the population. We denote by $\mathbb{B}(\mathcal{A}_i)$ the Borel σ -algebra on \mathcal{A}_i and by $d(\lambda, \lambda')$ the distance between two states measured with the respect to the weak topology. Each user’s payoff depends on opponents’ behavior through the distribution of opponents’ choices and of their strategies. The payoff of a user i in a

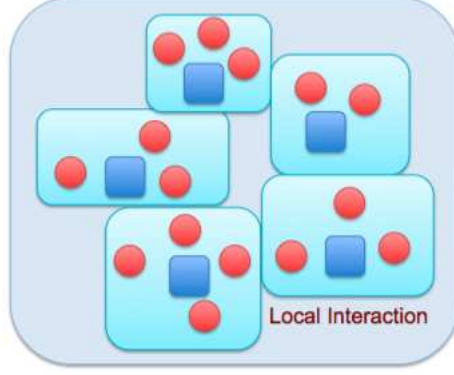


Figure 1: A population: distributed receivers and senders, represented by blue rectangles and red circles respectively.

local interaction with $(N - 1)$ other users is given as a function $u_i : \mathbb{R}^N \rightarrow \mathbb{R}$. The rate profile $\alpha \in \mathbb{R}^N$ must belong to a common capacity region $\mathcal{C} \subset \mathbb{R}^N$ defined by $2^N - 1$ linear inequalities. The expected payoff of a sender i transmitting at a rate a when the state of the population is $\mu \in \Delta(\mathcal{A}_i)$ is given by $F_i(a, \mu)$. The expected payoff for user i is

$$F_i(\lambda_i, \mu) := \int_{\alpha \in \mathcal{C}} u_i(\alpha) \lambda_i(d\alpha_i) \prod_{j \neq i} \mu(d\alpha_j).$$

The population state is subjected to the “mixed extension” of capacity constraints $\mathcal{M}(\mathcal{C})$. This will be discussed in Section 2.5 and will be made more precise later.

2.1 Local Interactions

Local interaction refers to the problem setting of one receiver and its uplink additive white Gaussian noise (AWGN) multiple access channel with N senders with coupled constraints (or actions). The signal at the receiver is given by $Y = \xi + \sum_{i=1}^N X_i$ where X_i is a transmitted signal of user i and ξ is a zero-mean Gaussian noise with variance σ_0^2 . Each user has an individual power constraint $\mathbb{E}(X_i^2) \leq P_i$ and the channel gain h_i . The optimal power allocation scheme is to transmit at the maximum power available, i.e. P_i , for each user. Hence, we consider the case in which maximum power is attained. The decisions of the users then consist of choosing their communication rates, and the receiver’s role is to decode, if possible. The capacity region is the set of all vectors $\alpha \in \mathbb{R}_+^N$ such that users $i \in \mathcal{N} := \{1, 2, \dots, N\}$ can reliably communicate at rate α_i , $i \in \mathcal{N}$. The capacity region \mathcal{C} for this channel is the set

$$\mathcal{C} = \left\{ \alpha \in \mathbb{R}_+^N \mid \sum_{i \in \Omega} \alpha_i \leq \log \left(1 + |\Omega| \frac{P_i h_i}{\sigma_0^2} \right), \forall \emptyset \subsetneq \Omega \subseteq \mathcal{N} \right\},$$

Example 1. (Example of capacity region with three users) In this example, we illustrate the capacity region with three users. Let $\alpha_1, \alpha_2, \alpha_3$ be the rates of the users and $P_i = P, h_i = h, \forall i = 1, 2, 3$.

Based on (1), we obtain a set of inequalities

$$\begin{cases} \alpha_1 \geq 0, \alpha_2 \geq 0, \alpha_3 \geq 0 \\ \alpha_i \leq \log \left(1 + \frac{Ph}{\sigma_0^2} \right), i = 1, 2, 3 \\ \alpha_i + \alpha_j \leq \log \left(1 + 2\frac{Ph}{\sigma_0^2} \right), i \neq j, i, j = 1, 2, 3. \\ \alpha_1 + \alpha_2 + \alpha_3 \leq \log \left(1 + 3\frac{Ph}{\sigma_0^2} \right) \end{cases} ,$$

or in the compact notation, $M_3 \gamma_3 \leq \zeta_3$, where

$$\gamma_3 := \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \in \mathbb{R}_+^3, \quad \zeta_3 := \begin{bmatrix} C_{\{1\}} \\ C_{\{2\}} \\ C_{\{3\}} \\ C_{\{1,2\}} \\ C_{\{1,3\}} \\ C_{\{2,3\}} \\ C_{\{1,2,3\}} \end{bmatrix}, \quad M_3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \in \mathbb{Z}^{7 \times 3}.$$

Note that M_3 is a totally unimodular matrix. By letting $Ph = 25, \sigma_0^2 = 0.1$, we sketch in Figure 2 the capacity region with three users.

The capacity region reveals a competitive nature of the interactions among senders: if a user i wants to communicate at a higher rate, one of the other users has to lower his rate; otherwise, the capacity constraint is violated. We let

$$r_{i,\Omega} := \log \left(1 + \frac{P_i h_i}{\sigma_0^2 + \sum_{i' \in \Omega, i' \neq i} P_{i'} h_{i'}} \right), i \in \mathcal{N}, \Omega \subseteq \mathcal{N}$$

denote the bound on the rate of a user when the signals of the $|\Omega| - 1$ other users are treated as noise.

Due to the noncooperative nature of the rate allocation, we can formulate the one-shot game

$$\Xi = \langle \mathcal{N}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}} \rangle,$$

where the set of users \mathcal{N} is the set of players, $\mathcal{A}_i, i \in \mathcal{N}$, is the set of actions, and $u_i, i \in \mathcal{N}$, are the payoff functions.

2.2 Payoffs

We define $u_i : \prod_{i=1}^N \mathcal{A}_i \rightarrow \mathbb{R}_+$ as follows:

$$u_i(\alpha_i, \alpha_{-i}) = \mathbb{1}_{\mathcal{C}}(\alpha) g_i(\alpha_i, \alpha_{-i}) \quad (1)$$

$$= \begin{cases} g_i(\alpha_i) & \text{if } (\alpha_i, \alpha_{-i}) \in \mathcal{C} \\ 0 & \text{otherwise} \end{cases}, \quad (2)$$

where $\mathbb{1}_{\mathcal{C}}$ is the indicator function; α_{-i} is a vector consisting of other players' rates, i.e., $\alpha_{-i} = [\alpha_1, \dots, \alpha_{i-1}, \alpha_{i+1}, \dots, \alpha_N]$ and u_i is a positive and strictly increasing function for each fixed α_{-i} . Since the game is subject to coupled constraints, the action set \mathcal{A}_i is coupled and dependent on other players' actions. Given the strategy profile α_{-i} of other players, the constrained action set \mathcal{A}_i is given by

$$\mathcal{A}_i(\alpha_{-i}) := \{\alpha_i \in [0, C_{\{i\}}], (\alpha_i, \alpha_{-i}) \in \mathcal{C}\} \quad (3)$$

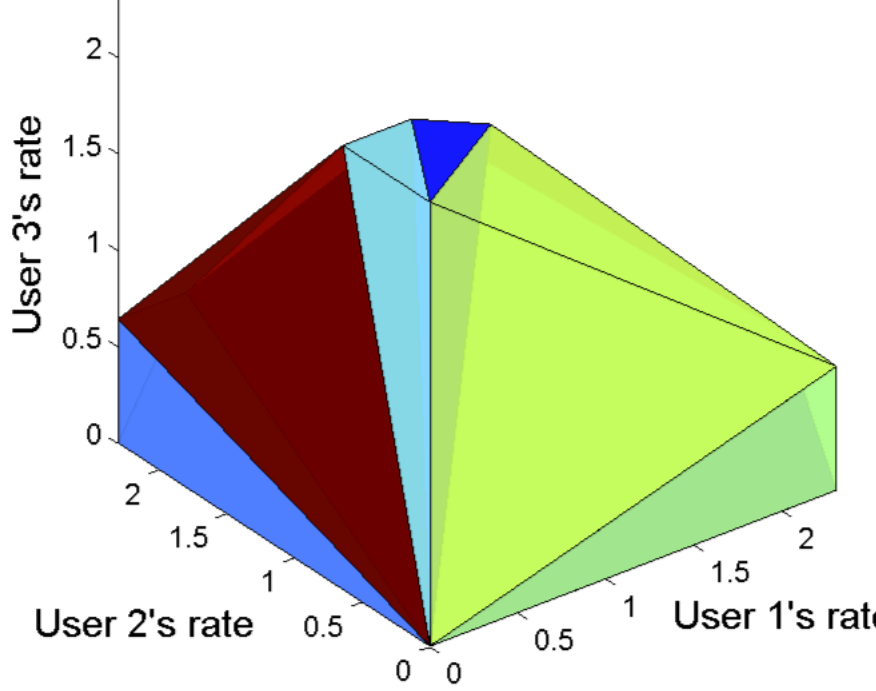


Figure 2: Capacity region with three users.

We then have an asymmetric game. The minimum rate that the user i can guarantee in the feasible regions is $r_{i,\mathcal{N}}$ which is different than $r_{j,\mathcal{N}}$.

Each user i maximizes $u_i(\alpha_i, \alpha_{-i})$ over the coupled constraint set. Owing to the monotonicity of the function g_i and the inequalities that define the capacity region, we obtain the following lemma.

Lemma 1. Let $\overline{BR}_i(\alpha_{-i})$ be the best reply to the strategy α_{-i} , defined by

$$\overline{BR}^i(\alpha_{-i}) = \arg \max_{y \in \mathcal{A}_i(\alpha_{-i})} u_i(y, \alpha_{-i}).$$

\overline{BR}_i is a non-empty single-valued correspondence (i.e. a standard function), and is given by

$$\max \left(r_{i,\mathcal{N}}, \min_{\Omega \in \Gamma_i} \left\{ C_\Omega - \sum_{k \in \Omega \setminus \{i\}} \alpha_k \right\} \right), \quad (4)$$

where $\Gamma_i = \{\Omega \in 2^{\mathcal{N}}, i \in \Omega\}$.

Proposition 1. *The set of Nash equilibria is*

$$\left\{ (\alpha_i, \alpha_{-i}) \mid \alpha^i \geq r_{i,\mathcal{N}}, \sum_{i \in \mathcal{N}} \alpha_i = C_{\mathcal{N}} \right\}.$$

All these equilibria are optimal in Pareto sense.

Proof. Let β be a feasible solution, i.e., $\beta \in \mathcal{C}$. If $\sum_{i=1}^N \beta_i < C_{\mathcal{N}} = \log \left(1 + \sum_{i \in \mathcal{N}} \frac{P_i h_i}{\sigma_0^2} \right)$, then at least one of the users can improve its rate (hence its payoff) to reach one of the faces of the capacity region. We now check the strategy profile on the face

$$\left\{ (\alpha_i, \alpha_{-i}) \mid \alpha^i \geq r_{i,\mathcal{N}}, \sum_{i=1}^N \alpha_i = C_{\mathcal{N}} \right\}.$$

If $\beta \in \left\{ (\alpha_i, \alpha_{-i}) \mid \alpha_i \geq r_{i,\mathcal{N}}, \sum_{i=1}^N \alpha_i = C_{\mathcal{N}} \right\}$, then from the Lemma 1, $\overline{BR}_i(\beta_{-i}) = \{\beta_i\}$. Hence, β is a strict equilibrium. Moreover, this strategy β is Pareto optimal because the rate of each user is maximized under the capacity constraint. These strategies are social welfare optimal if the total utility $\sum_{i=1}^N u_i(\alpha_i, \alpha_{-i}) = \sum_{i=1}^N g_i(\alpha_i)$ is maximized subject to constraints. \square

Note that the set of pure Nash equilibria is a convex subset of the capacity region.

2.3 Robust Equilibria and Efficiency Measures

2.3.1 Constrained Strong Equilibria and Coalition Proofness

An action profile in a local interaction between N senders is a constrained k -strong equilibrium if it is feasible and no coalition of size k can improve the rate transmissions of each of its members with respect to the capacity constraints. An action is a constrained strong equilibrium [18] if it is a constrained k -strong equilibrium for any size k . A strong equilibrium is then a policy from which no coalition (of any size) can deviate and improve the transmission rate of every member of the coalition (group of the simultaneous moves), while possibly lowering the transmission rate of users outside the coalition group. This notion of constrained strong equilibria¹ is very attractive because it is resilient against coalitions of users. Most of the games do not admit any strong equilibrium but in our case we will show that the multiple access channel game has several strong equilibria.

Theorem 1. *Any rate profile on the maximal face of the capacity region \mathcal{C} :*

$$Face_{\max}(\mathcal{C}) := \left\{ (\alpha_i, \alpha_{-i}) \in \mathbb{R}^N \mid \alpha_i \geq r_N, \sum_{i=1}^N \alpha_i = C_{\mathcal{N}} \right\},$$

is a constrained strong equilibrium.

Proof. We remark that if the rate profile α is not on the maximal face of the capacity region, then α is not resilient to deviation by a single user. Hence, α cannot be a constrained strong equilibrium. This says that a necessary condition for a rate profile to be a strong equilibrium is to be in the subset $Face_{\max}(\mathcal{C})$. We now prove that the condition: $\alpha \in Face_{\max}(\mathcal{C})$ is sufficient. Let $\alpha \in Face_{\max}(\mathcal{C})$. Suppose that k users deviate simultaneously from the rate profile α . Denote by Dev the set of users which deviate simultaneously (eventually by forming a coalition). The rate constraints of the deviants are

¹Note that the set of constrained strong equilibria is a subset of the set of Nash equilibria (by taking coalitions of size one) and any constrained strong equilibrium is Pareto optimal (by taking coalition of full size).

1. $\alpha'_i \geq 0, \forall i \in Dev,$
2. $\sum_{i \in \bar{\Omega}} \alpha'_i \leq C_{\bar{\Omega}}, \forall \bar{\Omega} \subseteq Dev,$
3. $\sum_{i \in \Omega \cap Dev} \alpha'_i \leq C_{\Omega} - \sum_{i \in \Omega, i \notin Dev} \alpha_i, \forall \Omega \subseteq \mathcal{N}, \Omega \cap Dev \neq \emptyset.$

In particular, for $\Omega = \mathcal{N}$, we have $\sum_{i \in Dev} \alpha'_i \leq C_{\mathcal{N}} - \sum_{i \notin Dev} \alpha_i$. The total rate of the deviants is bounded by $C_{\mathcal{N}} - \sum_{i \notin Dev} \alpha_i$, which is not controlled by the deviants. The deviants move to $(\alpha'_i)_{i \in Dev}$ with

$$\sum_{i \in Dev} \alpha'_i < C_{\mathcal{N}} - \sum_{i \notin Dev} \alpha_i.$$

Then, there exists i such that $\alpha_i > \alpha'_i$. Since g_i is non-decreasing, this implies that $g_i(\alpha_i) > g_i(\alpha'_i)$. The user i who is a member of the coalition Dev does not improve its payoff. If the rates of some of the deviants are increased, then the rates of some other users from coalition must decrease. If $(\alpha'_i)_{i \in Dev}$ satisfies

$$\sum_{i \in Dev} \alpha'_i = C_{\mathcal{N}} - \sum_{i \notin Dev} \alpha_i,$$

then some users in the coalition Dev have increased their rates compared with $(\alpha_i)_{i \in Dev}$ and some others in Dev have decreased their rates of transmission (because the total rate is the constant $C_{\mathcal{N}} - \sum_{i \notin Dev} \alpha_i$). The users in Dev with a lower rate $\alpha'_i \leq \alpha_i$ do not benefit by being a member of the coalition (Shapley criterion of membership of coalition does not hold). And this holds for any $\emptyset \subsetneq Dev \subseteq \mathcal{N}$. This completes the proof. \square

Corollary 1. *In the constrained rate allocation game, Nash equilibria and strong equilibria in pure strategies coincide.*

2.3.2 Constrained Potential Function for Local Interaction

Introduce the following function:

$$V(\alpha) = \mathbb{1}_{\mathcal{C}}(\alpha) \sum_{i=1}^N g_i(\alpha_i),$$

where $\mathbb{1}_{\mathcal{C}}$ is the indicator function of \mathcal{C} , i.e., $\mathbb{1}_{\mathcal{C}}(\alpha) = 1$ if $\alpha \in \mathcal{C}$ and 0 otherwise. The function V satisfies

$$V(\alpha) - V(\beta_i, \alpha_{-i}) = g_i(\alpha_i) - g_i(\beta_i), \forall \alpha, (\beta_i, \alpha_{-i}) \in \mathcal{C}.$$

If g_i is differentiable, then one has

$$\frac{\partial}{\partial \alpha_i} V(\alpha) = g'_i(\alpha_i) = \frac{\partial}{\partial \alpha_i} u_i$$

in the interior of the capacity region \mathcal{C} , and V is a constrained potential function [3] in pure strategies.

Corollary 2. *The local maximizers of V in \mathcal{C} are pure Nash equilibria. Global maximizers of V in \mathcal{C} are both constrained strong equilibria and social optima for the local interaction.*

2.3.3 Strong Price of Anarchy

Throughout this subsection, we assume that the functions g_i are the identity function, i.e., $g_i(x) = id(x) := x$. One metric used to measure how much the performance of decentralized systems is affected by the selfish behavior of its components is the *price of anarchy*. We present a similar price for strong equilibria under the coupled rate constraints. This notion of Price of Anarchy can be seen as an *efficiency metric* that measures the *price of selfishness* or decentralization and has been extensively used in the context of congestion games or routing games where typically users have to minimize a cost function [41, 42]. In the context of rate allocation in the multiple access channel, we define an equivalent measure of price of anarchy for rate maximization problems. One of the advantages of a strong equilibrium is that it has the potential to reduce the distance between the optimal solution and the solution obtained as an outcome of selfish behavior, typically in the case where the capacity constraint is violated at each time. Since the constrained rate allocation game has strong equilibria, we can define the strong price of anarchy, introduced in [12], as the ratio between the payoff of the worst constrained strong equilibrium and the social optimum value which C_N .

Theorem 2. *The strong price of anarchy of the constrained rate allocation game is 1 for $g_i(x) = x$.*

Note that for $g_i \neq id$, the CSPoA can be less than one. However, the optimistic price of anarchy of the *best constrained equilibrium*, also called *price of stability* [13], is one for any function g_i i.e the efficiency of “best” equilibria is 100%.

2.4 Selection of Pure Equilibria

We have shown in the previous sections that our rate allocation game has a continuum of pure Nash equilibria and strong equilibria. We address now the problem of selecting one equilibrium which has certain desirable properties: the normalized pure Nash equilibrium, introduced in [29]; see also [31, 20, 23]. We introduce the problem of constrained maximization faced by each user when the other rates are at the maximal face of the polytope \mathcal{C} :

$$\max_{\alpha} \quad u_i(\alpha) \tag{5}$$

$$\text{s.t.} \quad \alpha_1 + \dots + \alpha_N = C_N \tag{6}$$

for which the corresponding Lagrangian for user i is given by

$$L_i(\alpha, \zeta) = u_i(\alpha) - \zeta_i \left(\sum_{i=1}^N \alpha_i - C_N \right).$$

From Karush-Kuhn-Tucker optimality conditions, it follows that there exists $\zeta \in \mathbb{R}^N$ such that

$$g'_i(\alpha_i) = \zeta_i, \quad \sum_{i=1}^N \alpha_i = C_N.$$

For a fixed vector ζ with identical entries, define the normal form game $\Gamma(\zeta)$ with N users, where actions are taken as rates and the payoffs given by $L(\alpha, \zeta)$. A normalized equilibrium is an equilibrium of the game $\Gamma(\zeta^*)$ where ζ^* is normalized into the form $\zeta_i^* = \frac{c}{\tau_i}$, $c > 0, \tau_i > 0$. We now have the following result due to Goodman [20] which implies Rosen’s condition on uniqueness for strict concave games.

Theorem 3. Let u_i be a smooth and strictly concave function in α_i , each u_i be convex in α_{-i} , and there exist some ζ such that the weighted non-negative sum of the payoffs $\sum_{i=1}^N \zeta_i u_i(\alpha)$ is concave in α . Then, the matrix $G(\alpha, \zeta) + G^T(\alpha, \zeta)$ is negative definite (which implies uniqueness) where $G(\alpha, \zeta)$ is the Jacobian with respect to α of

$$h(\alpha, \zeta) := [\zeta_1 \nabla_1 u_1(\alpha), \zeta_2 \nabla_2 u_2(\alpha), \dots, \zeta_N \nabla_N u_N(\alpha)]^T$$

and G^T is the transpose of the matrix G .

This now leads to the following corollary for our problem.

Corollary 3. If g_i are non-decreasing strictly concave functions, then the rate allocation game has a unique normalized equilibrium which corresponds to an equilibrium of the normal form game with payoff $L(\alpha, \zeta^*)$ for some ζ^* .

2.5 Stability and Dynamics

In this subsection, we study the stability of equilibria and several classes of evolutionary game dynamics under a symmetric case, i.e., $P_i = P, h_i = h, g_i = g, \mathcal{A}_i = \mathcal{A}, \forall i \in \mathcal{N}$. We will drop subscript index i where appropriate. We show that the associated evolutionary game has a unique pure constrained evolutionary stable strategy.

Proposition 2. The collection of rates $\alpha = \left(\frac{C_N}{N}, \dots, \frac{C_N}{N}\right)$, i.e. the distribution of Dirac concentrated on the rate $\frac{C_N}{N}$, is the unique symmetric pure Nash equilibrium.

Proof. Since the constrained rate allocation game is symmetric, there exists a symmetric (pure or mixed) Nash equilibrium. If such an equilibrium exists in pure strategies, each user transmits with the same rate r^* . It follows from Proposition 1 of Section 2.2, and the bound $r_N \leq \frac{C_N}{N}$ that r^* satisfies $Nr^* = C_N$ and r^* is feasible. \square

Since the set of feasible actions is convex, we can define convex combination of rates in the set of the feasible rates. For example, $\epsilon \alpha' + (1 - \epsilon)\alpha$ is a feasible rate if α' and α are feasible. The symmetric rate profile (r, r, \dots, r) is feasible if and only if $0 \leq r \leq r^* = \frac{C_N}{N}$. We say that the rate r is a constrained evolutionarily stable strategy (ESS) if it is feasible and for every mutant strategy $mut \neq \alpha$ there exists $\epsilon_{mut} > 0$ such that

$$\begin{cases} r_\epsilon := \epsilon mut + (1 - \epsilon)r \in \mathcal{C} & \forall \epsilon \in (0, \epsilon_{mut}) \\ u(r, r_\epsilon, \dots, r_\epsilon) > u(mut, r_\epsilon, \dots, r_\epsilon) & \forall \epsilon \in (0, \epsilon_{mut}) \end{cases}$$

Theorem 1. The pure strategy $r^* = \frac{C_N}{N}$ is a constrained evolutionary stable strategy.

Proof. Let $mut \leq r^*$. The rate $\epsilon mut + (1 - \epsilon)r^*$ is feasible implies that $mut \leq r^*$ (because r^* is the maximum symmetric rate achievable). Since $mut \neq r^*$, mut is strictly lower than r^* . By monotonicity of the function g , one has $u(r^*, \epsilon mut + (1 - \epsilon)r^*) > u(mut, \epsilon mut + (1 - \epsilon)r^*)$, $\forall \epsilon$. This completes the proof. \square

2.5.1 Symmetric Mixed Strategies

Define the mixed capacity region $\mathcal{M}(\mathcal{C})$ as the set of measures profile $(\mu_1, \mu_2, \dots, \mu_N)$ such that

$$\int_{\mathbb{R}_+^{|\Omega|}} \left(\sum_{i \in \Omega} \alpha_i \right) \prod_{i \in \Omega} \mu_i(d\alpha_i) \leq C_\Omega, \quad \forall \Omega \subseteq \mathcal{N}.$$

Then, the payoff of the action $a \in \mathbb{R}_+$ satisfying $(a, \lambda, \dots, \lambda) \in \mathcal{M}(\mathcal{C})$ can be defined as

$$F(a, \mu) = \int_{[0, \infty[^{N-1}} u(a, b_2, \dots, b_N) \nu_{N-1}(db), \quad (7)$$

where $\nu_k = \bigotimes_1^k \mu$ is the product measure on $[0, \infty[^k$. The constraint set becomes the set of probability measures on \mathbb{R}_+ such that

$$0 \leq \mathbb{E}(\mu) := \int_{\mathbb{R}_+} \alpha_i \mu(d\alpha_i) \leq \frac{C_N}{N} < C_{\{1\}}.$$

Lemma 2. *The payoff can be obtained as follows:*

$$F(a, \mu) = \mathbb{1}_{[0, C_N - (N-1)\mathbb{E}(\mu)]} \times g(a) \times \int_{b \in \mathcal{D}_a} \nu_{N-1}(db) = \mathbb{1}_{[0, C_N - (m-1)\mathbb{E}(\mu)]} \times g(a) \nu_{N-1}(\mathcal{D}_a),$$

where $\mathcal{D}_a = \{(b_2, \dots, b_N) \mid (a, b_2, \dots, b_N) \in \mathcal{C}\}$.

Proof. If the rate does not satisfy the capacity constraints, then the payoff is 0. Hence the *rational* rate for user i is lower than $C_{\{i\}}$. Fix a rate $a \in [0, C_{\{i\}}]$. Let $D_\Omega^a := C_\Omega - a\delta_{\{1 \in \Omega\}}$. Then, a necessary condition to have a non-zero payoff is $(b_2, \dots, b_N) \in \mathcal{D}_a$, where $\mathcal{D}_a = \{(b_2, \dots, b_N) \in \mathbb{R}_+^{N-1}, \sum_{i \in \Omega, i \neq 1} b_i \leq D_\Omega^a, \Omega \subseteq 2^N\}$. Thus, we have

$$\begin{aligned} F(a, \mu) &= \int_{\mathbb{R}_+^{N-1}} u(a, b_2, \dots, b_N) \nu_{N-1}(db) \\ &= \int_{b \in \mathbb{R}_+^{N-1}, (a, b) \in \mathcal{C}} g(a) \nu_{N-1}(db) \\ &= \mathbb{1}_{[0, C_N - (N-1)\mathbb{E}(\mu)]} g(a) \times \int_{b \in \mathcal{D}_a} \nu_{N-1}(db) \end{aligned}$$

□

2.5.2 Constrained Evolutionary Game Dynamics

The class of evolutionary games in large population provides a simple framework for describing strategic interactions among large numbers of users. In this subsection we turn to modeling the behavior of the users who play them. Traditionally, predictions of behavior in game theory are based on some notion of equilibrium, typically Cournot equilibrium, Bertrand equilibrium, Nash equilibrium, Stackelberg solution, Wardrop equilibrium or some refinement thereof. These notions require the assumption of equilibrium knowledge, which posits that each user correctly anticipates how his opponents will act. The equilibrium knowledge assumption is too strong and is difficult to justify in particular in contexts with large numbers of users. As an alternative to the equilibrium approach, we propose an explicitly dynamic updating choice, a procedure in which users myopically update their behavior in response to their current strategic environment. This dynamic procedure does not assume the automatic coordination of users' actions and beliefs, and it can derive many specifications of users' choice procedures. These procedures are specified formally by defining a revision of rates called *revision protocol* [30]. A revision protocol takes current payoffs and current mean rate and maps to conditional switch rates which describe how frequently users in some class playing rate α who are considering switching rates switch to strategy α' . Revision protocols are flexible enough to incorporate a wide variety of paradigms, including ones based on imitation, adaptation, learning, optimization, etc.

We use here a class of continuous evolutionary dynamics. We refer to [33, 14, 34] for evolutionary game dynamics with or without time delays. The continuous-time evolutionary game dynamics on the measure space $(\mathcal{A}, \mathcal{B}(\mathcal{A}), \mu)$ is given by

$$\dot{\lambda}_t(E) = \int_{a \in E} V(a, \lambda_t) \mu(da) \quad (8)$$

where

$$V(a, \lambda_t) = K \left[\int_{x \in \mathcal{A}} \beta_a^x(\lambda_t) \lambda_t(dx) - \int_{x \in \mathcal{A}} \beta_x^a(\lambda_t) \lambda_t(dx) \right],$$

and β_a^x represents the rate of mutation from x to a , and K is a growth parameter. $\beta_a^x(\lambda_t) = 0$ if (x, λ_t) or (a, λ_t) is not in the (mixed) capacity region, E is a μ -measurable subset of \mathcal{A} . At each time t , probability measure λ_t satisfies $\frac{d}{dt} \lambda_t(\mathcal{A}) = 0$. We examine the following classes of evolutionary game dynamics, namely, Brown-von Neumann-Nash dynamics, Smith dynamics and replicator dynamics, where $F(a, \lambda_t)$ is the payoff in (7) as defined in the previous subsection.

RD-1: Constrained Brown-von Neumann-Nash dynamics.

$$\beta_a^x(\lambda_t) = \begin{cases} \max(F(a, \lambda_t) - \int_x F(x, \lambda_t) dx, 0) & \text{if } (a, \lambda_t), (x, \lambda_t) \in \mathcal{M}(\mathcal{C}), \\ 0 & \text{otherwise.} \end{cases}$$

RD-2: Constrained Replicator Dynamics.

$$\beta_a^x(\lambda_t) = \begin{cases} \max(F(a, \lambda_t) - F(x, \lambda_t), 0) & \text{if } (a, \lambda_t), (x, \lambda_t) \in \mathcal{M}(\mathcal{C}) \\ 0 & \text{otherwise.} \end{cases}$$

RD-3: Constrained θ -Smith Dynamics.

$$\beta_a^x(\lambda_t) = \begin{cases} \max(F(a, \lambda_t) - F(x, \lambda_t), 0)^\theta & \text{if } (a, \lambda_t), (x, \lambda_t) \in \mathcal{M}(\mathcal{C}) \\ 0 & \text{otherwise.} \end{cases}, \quad \theta \geq 1$$

A common property that applies to all these dynamics is that the set of Nash equilibria is a subset of rest points (stationary points) of the evolutionary game dynamics. Here we extend the concepts of these dynamics to evolutionary games with a continuum action space and coupled constraints, and more than two-users interactions. The counterparts of these results in discrete action space can be found in [21, 30].

Theorem 2. *Any Nash equilibrium of the game is a rest point of the following evolutionary game dynamics: constrained Brown-von Neumann-Nash, generalized Smith dynamics, and replicator dynamics. In particular, the evolutionary stable strategies set is a subset of the rest points of these constrained evolutionary game dynamics.*

Proof. It is clear for pure equilibria by using the revision protocols β of these dynamics. Let λ be an equilibrium. For any rate a in the support of λ , $\beta_x^a = 0$ if $F(x, \lambda) \leq F(a, \lambda)$. Thus, if λ is an equilibrium, the difference between the microscopic inflow and outflow is $V(a, \lambda) = 0$, given that a is the support of the measure λ . \square

Let λ be a finite Borel measure on $[0, C_{\{i\}}]$ with full support. Suppose g is continuous on $[0, C_{\{i\}}]$. Then, λ is a rest point of the BNN dynamics if and only if λ is a symmetric Nash equilibrium. Note that the choice of topology is an important issue when defining dynamics convergence and stability of the dynamics. The most used in this area is the topology of the weak convergence to measure closeness of two states of the system. Different distances (Prohorov metric, metric on bounded and Lipschitz continuous functions on \mathcal{A}) have been proposed. We refer the reader to [26], and the references therein for more details on *evolutionary robust strategy* and stability notions.

2.6 Correlated Equilibrium

In this subsection, we analyze constrained correlated equilibria of multiple access (MAC) games. Building upon the signaling in the one-shot game, we formulate a system of evolutionary MAC games with evolutionary game dynamics that describe the evolution of signaling, beliefs, rate control and channel selection, respectively.

We focus on correlated equilibrium in the single-receiver case. Correlated strategies are based on signaling structures before making decisions on rates. Different scenarios (with or without mediator, virtual mediator, cryptographic multi-stage signaling structure) have been proposed in the literature [15, 39, 16, 17].

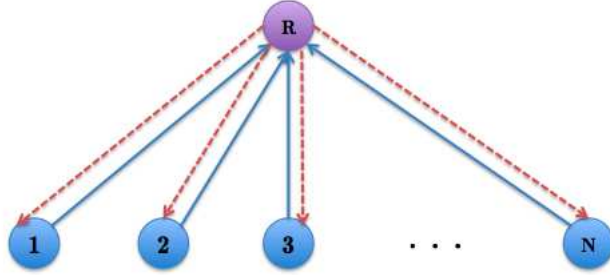


Figure 3: Signaling between multiple senders and a receiver

In Figure 3, we illustrate the signaling between multiple transmitters and one receiver. The receiver can act as a signaling device to mediate the behaviors of the transmitters. The correlated equilibrium has a strong connection with cryptography in that the private signal sent to the users can be realized by the coding and decoding in the network [39].

Let \mathcal{B} be the set of signals $\beta = [\beta_i, \beta_{-i}] \in \mathbb{R}^N$. The values β from the set of signals need to be in the feasible set $\mathcal{C} \subset \mathbb{R}^N$. Let $\mu \in \Delta\mathcal{B}$ be a probability measure over the set \mathcal{B} . A constrained correlated equilibrium (CCE) μ^* need to satisfy the following set of inequalities,

$$\int d\mu^*(\beta_i, \beta_{-i}) [u_i(\alpha_i, \alpha_{-i} | \beta_i) - u_i(\alpha'_i, \alpha_{-i} | \beta_i)] \geq 0, \forall i \in \mathcal{N}, \alpha'_i \in \mathcal{A}_i(\alpha_{-i}).$$

Define a rule of assignment of user i as a map its signals to its action's set $\bar{r}_i : \beta_i \mapsto \alpha_i$. A CCE is then characterized by

$$\int d\mu^*(\beta) [u_i(\alpha_i, \alpha_{-i} | \beta_i) - u_i(r_i(\beta_i), \alpha_{-i})] \geq 0, \forall i \in \mathcal{N}, \forall r_i \text{ such that } \bar{r}_i(\cdot) \in \mathcal{A}_i(\alpha_{-i}). \quad (9)$$

Theorem 3. *The set of constrained pure Nash equilibria of the MISO game is given by*

$$\text{max-face}(\mathcal{C}) = \left\{ (\alpha_1, \dots, \alpha_N) \mid \alpha_i \geq 0, \sum_{k \in \mathcal{N}} \alpha_k = C_{\mathcal{N}} \right\}$$

We can characterize the CCE using the above results as follows.

Lemma 3. *Any mixture of constrained pure Nash equilibria of the MISO game is a constrained correlated equilibrium.*

Note that the set of constrained correlated equilibria is bigger than the set of constrained Nash equilibria. For example, in a two-user case, the distribution $\frac{1}{2}\delta_{(r_1, C_{\{1,2\}} - r_1)} + \frac{1}{2}\delta_{(C_{\{1,2\}} - r_2, r_2)}$ is different than the Dirac distribution $\delta_{(\frac{r_1 + C_{\{1,2\}} - r_2}{2}, \frac{r_2 + C_{\{1,2\}} - r_1}{2})}$.

Proof. Let μ be a probability distribution over some constrained pure equilibria. Then, $\mu \in \Delta(\text{max-face}(\mathcal{C}))$. For any β such that $\mu(\beta) \neq 0$, one has

$$[u_i(\alpha_i, \alpha_{-i} \mid \beta_i) - u_i(\bar{r}_i(\beta_i), \alpha_{-i})] \geq 0$$

for any measurable function $\bar{r}_i : [0, C_{\{i\}}] \rightarrow [0, C_{\{i\}}]$. Thus, μ is a constrained correlated equilibrium. □

Corollary 4. *Any convex combination of extreme point of the convex compact set*

$$\text{max-face}(\mathcal{C}) = \left\{ \alpha = (\alpha_1, \dots, \alpha_N) \mid \alpha_i \geq r_i, \sum_{k \in \mathcal{N}} \alpha_k = C_{\mathcal{N}} \right\}$$

is a constrained correlated equilibrium. Moreover any probability distribution over the maximal face of the capacity region $\text{max-face}(\mathcal{C})$ is a correlated constrained equilibrium distribution.

3 Hybrid AWGN Multiple Access Control

In this section, we extend the single receiver case to one with multiple receivers. Multi-input and multi-output (MIMO) channel access game has been studied in the context of power allocation and control. For instance, the authors in [6] formulate a two-player zero-sum game where the first player is the group of transmitters and the second one is the set of MIMO sub-channels. In [5], the authors formulate an N -person non-cooperative power allocation game and study its equilibrium under two different decoding schemes.

Here, we formulate a hybrid multiple access game where users are allowed to select their rates and channels under capacity constraints. We first obtain general results on the existence of the equilibrium and methods to characterize it. In addition, we investigate long-term behavior of the strategies and apply evolutionary game dynamics to both rates and channel selection probabilities. We show that G-function based dynamics is appropriate for our hybrid model by viewing the channel selection probabilities as strategies that determine of fitness of rate selection. Using the generalized Smith dynamics for channel selection, we are able to build an overall hybrid evolutionary dynamics for the static model. Based on simulations, we confirm the validity of these proposed dynamics and the correspondence between the rest point of the dynamics and the Nash equilibrium.

3.1 Hybrid Model with Rate Control and Channel Selection

In this subsection, we establish a model for multiple users and multiple receivers. Each user needs to decide the rate at which to transmit and the channel to pick. We formulate a game $\Xi = \langle \mathcal{N}, (\mathcal{A}_i)_{i \in \mathcal{N}}, (\bar{U}_i)_{i \in \mathcal{N}} \rangle$, in which the decision variable is (α_i, \mathbf{p}_i) , and $\mathbf{p}_i = [p_{ij}]_{j \in \mathcal{J}}$ is a J -dimensional vector, where p_{ij} is the probability of user $i \in \mathcal{N}$ to choose channel $j \in \mathcal{J}$ and p_{ij} needs to satisfy the probability measure constraints

$$\sum_{j \in \mathcal{J}} p_{ij} = 1, p_{ij} \geq 0, \forall i \in \mathcal{N} \tag{10}$$

The game $\bar{\Xi}$ is *asymmetric* in the sense that the strategy sets of users are different and the payoffs are not symmetric.

Let $C_{j,\Omega} := \log \left(1 + \sum_{i \in \Omega} \frac{P_{ij} h_{ij}}{\sigma_0^2} \right)$ be the capacity for a subset $\Omega \subseteq \mathcal{N}$ of users at receiver $j \in \mathcal{J}$ and $r_{ij,\Omega} := \log \left(1 + \frac{P_{ij} h_{ij}}{\sigma_0^2 + \sum_{i' \in \Omega, i' \neq i} P_{i'j} h_{i'j}} \right)$ the bound rate of a user i when the signals of the $|\Omega| - 1$ other users are treated as noise at receiver j . Each receiver j has a capacity region $\mathcal{C}(j)$ given by

$$\mathcal{C}(j) = \left\{ (\alpha, \mathbf{p}_j) \in [0, 1]^N \times \mathbb{R}_+^N \mid \sum_{i \in \mathcal{N}} \alpha_i p_{ij} \leq C_{j,\Omega}, \forall \emptyset \subset \Omega_j \subseteq \mathcal{N}, j \in \mathcal{J} \right\}, \quad (11)$$

The expected payoff function $\bar{U}_i : \prod_{i=1}^N \mathcal{A}_i \rightarrow \mathbb{R}_+$ of the game is given by

$$\bar{U}_i(\alpha_i, \mathbf{p}_i, \alpha_{-i}, \mathbf{p}_{-i}) = \mathbb{E}_{\mathbf{p}_i} [u_{ij}(\alpha, \mathbf{P})] = \sum_{j \in \mathcal{J}} p_{ij} u_{ij}(\alpha, \mathbf{P}), \quad (12)$$

where $\alpha = (\alpha_i, \alpha_{-i}) \in \mathbb{R}_+^N$ and $\mathbf{P} \in [0, 1]^{N \times J} = (\mathbf{p}_i, \mathbf{p}_{-i})$, with $\mathbf{p}_i \in [0, 1]^J$, $\mathbf{p}_{-i} \in [0, 1]^{(N-1) \times J}$. Assume that the utility u_{ij} of a user i transmitting to receiver j is only dependent on the user himself and is described by a positive and strictly increasing function $g_i : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, i.e., $u_{ij} = g_i, \forall j \in \mathcal{J}$, when capacity constraints are satisfied.

With the presence of coupled constraints (11) from each receiver and probability measure constraint (10), each sender has his individual optimization problem (IOP) given as follows.

$$\begin{aligned} \max_{\mathbf{p}_i, \alpha_i} \quad & \sum_{j \in \mathcal{J}} p_{ij} g_i(\alpha_i p_{ij}) \\ \text{s.t.} \quad & \sum_{j \in \mathcal{J}} p_{ij} = 1, \forall i \in \mathcal{N} \\ & p_{ij} \geq 0, \forall i \in \mathcal{N}, j \in \mathcal{J} \\ & (\alpha, \mathbf{p}_j) \in \mathcal{C}(j), \forall j \in \mathcal{J} \end{aligned}$$

Denote the feasible set of (IOP) by $\mathcal{F} = \mathcal{F}_1 \times \mathcal{F}_2$, where

$$\mathcal{F}_1 = \{ \alpha \in \mathbb{R}_+^N \mid (\alpha, \mathbf{P}) \in \cap_{j \in \mathcal{J}} \mathcal{C}(j), \mathbf{P} \in \mathcal{F}_2 \}, \quad (13)$$

$$\mathcal{F}_2 = \left\{ \mathbf{P} \in \mathbb{R}^{N \times J} \mid \sum_{j \in \mathcal{J}} p_{ij} = 1, p_{ij} \geq 0, \forall i \in \mathcal{N}, j \in \mathcal{J} \right\}. \quad (14)$$

The action set of each user can thus be described by

$$\mathcal{A}_i(\alpha_{-i}, \mathbf{p}_{-i}) = \{ (\alpha_i, \alpha_{-i}) \in \mathcal{F}_1, (\mathbf{p}_i, \mathbf{p}_{-i}) \in \mathcal{F}_2 \}. \quad (15)$$

3.1.1 An Example

Suppose we have three users and three receivers, that is, $\mathcal{N} = \{1, 2, 3\}$ and $\mathcal{J} = \{1, 2, 3\}$. The capacity region at receiver 1 is given by

$$\mathcal{C}(1) = \left\{ \begin{array}{l} \alpha_i \geq 0, \ i = 1, 2, 3 \\ p_{11}\alpha_1 \leq \log(1 + \frac{P_1 h_1}{\sigma_0^2}) \\ p_{21}\alpha_2 \leq \log(1 + \frac{P_2 h_2}{\sigma_0^2}) \\ p_{31}\alpha_3 \leq \log(1 + \frac{P_3 h_3}{\sigma_0^2}) \\ p_{11}\alpha_1 + p_{21}\alpha_2 \leq \log(1 + \frac{P_1 h_1 + P_2 h_2}{\sigma_0^2}) \\ p_{11}\alpha_1 + p_{31}\alpha_3 \leq \log(1 + \frac{P_1 h_1 + P_3 h_3}{\sigma_0^2}) \\ p_{21}\alpha_2 + p_{31}\alpha_3 \leq \log(1 + \frac{P_2 h_2 + P_3 h_3}{\sigma_0^2}) \\ p_{11}\alpha_1 + p_{21}\alpha_2 + p_{31}\alpha_3 \leq \log(1 + \frac{P_1 h_1 + P_2 h_2 + P_3 h_3}{\sigma_0^2}) \\ 0 \leq p_{i1} \leq 1, \ i = 1, 2, 3 \end{array} \right\}.$$

This can be written into

$$\mathcal{C}(1) = \left\{ \mathbf{p}_1 = \begin{bmatrix} p_{11} \\ p_{21} \\ p_{31} \end{bmatrix} \in [0, 1]^3, \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \in \mathbb{R}_+^3 \mid M_3 \begin{bmatrix} p_{11}\alpha_1 \\ p_{21}\alpha_2 \\ p_{31}\alpha_3 \end{bmatrix} \leq \begin{bmatrix} C_{1,\{1\}} \\ C_{1,\{2\}} \\ C_{1,\{3\}} \\ C_{1,\{1,2\}} \\ C_{1,\{1,3\}} \\ C_{1,\{2,3\}} \\ C_{1,\{1,2,3\}} \end{bmatrix} \right\},$$

where $C_{1,\Omega} = \log\left(1 + \sum_{i \in \Omega} \frac{P_i h_{i1}}{\sigma_0^2}\right)$ and M_3 is a totally unimodular matrix: $M_3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Capacity regions at receivers 2 and 3 can be obtained in a similar way.

3.2 Characterization of Constrained Nash Equilibria

In this subsection, we characterize the Nash equilibria of the defined game $\bar{\Xi}$ under the given capacity constraint. We use the following theorem to prove the existence of Nash equilibrium for the case where the rates are predetermined; this result is then used to solve the game for the case when both the rates and the connection probabilities are (joint) decision variables.

Theorem 4. (*Başar & Olsder, [38]*) Let $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2 \cdots \times \mathcal{A}_N$ be a closed, bounded and convex subset of \mathbb{R}^N , and for each $i \in \mathcal{N}$, the payoff functional $\bar{U}_i : \mathcal{A} \rightarrow \mathbb{R}$ be jointly continuous in \mathcal{A} and concave in a_i for every $a_j \in \mathcal{A}_j, j \in \mathcal{N}, j \neq i$. Then, the associated N -person nonzero-sum game admits a Nash equilibrium in pure strategies.

Applying Theorem 4, we have the following results immediately.

Proposition 3. Suppose $\alpha_i, i \in \mathcal{N}$, are predetermined feasible rates. Let feasible set \mathcal{F} be closed, bounded and convex. If g_i in (IOP) are continuous on \mathcal{F} and concave in \mathbf{p}_i (without the assumption of being positive and strictly increasing), the expected payoff functions $\bar{U}_i : \mathbb{R}_+^N \times [0, 1]^{N \times J} \rightarrow \mathbb{R}$ are concave in \mathbf{p}_i and continuous on \mathcal{F} , then the static game admits a Nash equilibrium.

The existence result in Proposition 3 only captures the case where the rates α_i are predetermined, and relies on the convexity requirement of the utility functions g_i . We can actually obtain a stronger existence result by observing that the formulated game $\bar{\Xi}$ is a potential game with a potential function given by

$$\Psi(\alpha, \mathbf{P}) = \sum_i f_i(\alpha_i, \mathbf{p}_i) = \sum_i \sum_j p_{ij} g_i(\alpha_i p_{ij}), \quad (16)$$

where $f_i = \sum_j p_{ij} g_i(\alpha_i p_{ij})$, the expected payoff to user i .

Note that the feasible set \mathcal{F} is generally nonempty and bounded. We can conclude the existence result in Proposition 4 of NE from this observation.

Proposition 4. The hybrid rate control game $\bar{\Xi}$ admits a Nash equilibrium.

Proof. Let us formulate a centralized optimization problem (COP) as follows.

$$\begin{aligned} \max_{\alpha, \mathbf{P}} \quad & \Psi(\alpha, \mathbf{P}) \\ \text{s.t.} \quad & (\alpha, \mathbf{P}) \in \mathcal{F} \end{aligned}$$

Using the result in [3], we can conclude that if there exists a solution to (COP), then there exists a Nash equilibrium to the game $\bar{\Xi}$. Since \mathcal{F} is compact and nonempty, and the objective function is continuous, there exists a solution to (COP) and thus a Nash equilibrium to the game. \square

The problem above is generally not convex and uniqueness of the Nash equilibrium may not be guaranteed. However, we still can further characterize the Nash equilibrium through the following propositions.

Proposition 5. Let $\beta_{ij} := \alpha_i p_{ij}$. Without predetermining α , suppose that $(\mathbf{p}_{-i}, \alpha_{-i})$ is feasible. A best response strategy at receiver $j \in \mathcal{J}$ for user i must satisfy

$$0 \leq p_{ij} \alpha_i \leq C_{j, \Omega_j} - \sum_{k \neq i} \alpha_k p_{kj}, \forall \Omega_j \quad (17)$$

where $\Omega_j := \{\Omega \in 2^{\mathcal{N}} \mid i' \in \Omega, p_{i'j} > 0\}$ is the set of users transmitting to receiver j . Since g_i is a non-decreasing function, the best correspondence at j is

$$\beta_{ij}^* = \alpha_i^* p_{ij}^* = \max \left(r_{ij, \mathcal{N}}, \min_{\Omega_j} \left(C_{j, \Omega_j} - \sum_{i' \neq i} \alpha_{i'} p_{i'j} \right) \right), \quad (18)$$

where $r_{ij, \mathcal{N}}$ is the bound on the rate of user i when signals of $|\mathcal{N}| - 1$ other users are treated as noise.

Proof. The proof is immediate by observing that the rate of user i at receiver j must satisfy (17) due to the coupled constraints. Thus, the maximum rate that user i can use to transmit to receiver j without violating the constraints is clearly the minimum of $C_{j, \Omega_j} - \sum_{i' \neq i} \alpha_{i'} p_{i'j}$ over all Ω_j . Since the payoff is a non-decreasing function, the best response for i at receiver j is given by (18). \square

Proposition 6. Let $K_i^* = \arg \max_{j \in \mathcal{J}} g_{ij}(\beta_{ij})$. If $K_i^* = \{k^*\}$ is a singleton, then the best reply for user i is to choose

$$\begin{cases} p_{ij} = 1 & \text{if } j = k_i^*, \\ p_{ij} = 0 & \text{otherwise,} \end{cases}$$

and we can determine α_i by $\alpha_i = \frac{\beta_{ik^*}}{p_{ik^*}}$.

If $|K_i^*| \geq 2$, then the best response correspondence is

$$\begin{cases} \mathbf{p}_i \in \Delta(K_i^*) & \text{if } j \in K_i^*, \\ 0 & \text{otherwise.} \end{cases}$$

We can determine α_i from β_{ij} by $\alpha_i = \sum_{j \in K_i^*} \beta_{ij}$.

Proof. Since the expected utility is given in the form of

$$U_i(\alpha_i, \mathbf{p}_i, \alpha_{-i}, \mathbf{p}_{-i}) = \mathbb{E}_{\mathbf{p}_i}[u_{ij}(\alpha_i p_{ij})],$$

the expected utility under best response is $U_i = \mathbb{E}_{\mathbf{p}_i}[u_{ij}(\beta_{ij})]$. If for a singleton k^* such that $k^* = \arg \max_{i \in \mathcal{N}} g_{ij}(\beta_{ij})$, we can assign all the weight $p_{ik^*} = 1$ to maximize the expected utility. Since $\beta_{ik^*} = \alpha_i p_{ik^*}$, then $\alpha = \beta_{ik^*}/p_{ik^*} = \beta_{ik^*}$. If the set K^* is not a singleton, without loss of generality, we can pick two indices $j, j' \in K^*$ such that $\beta_{ij} = \alpha_i p_{ij}$ and $\beta_{ij'} = \alpha_i p_{ij'}$, leading to $u_{ij}(\beta_{ij}) = u_{ij'}(\beta_{ij'})$. Since the utilities to transmit using j and j' are the same, we can assign arbitrary (two-point) distribution, p_{ij} and $p_{ij'}$ over them, with $p_{ij} + p_{ij'} = 1$. Therefore, $\beta_{ij} + \beta_{ij'} = \alpha_i(p_{ij} + p_{ij'}) = \alpha_i$. \square

3.3 Multiple Access Evolutionary Games

Interactions among users are dynamic and the users can update their rates and channel selection with respect to their payoffs and the known coupled constraints. Such a dynamic process can generally be modeled by either an evolutionary process, a learning process or a trial-and-error updating process. In classical game theory, the focus is on strategies that optimize payoffs to the players while, in evolutionary game theory, the focus is on strategies that will persist through time. In this subsection, we formulate evolutionary games dynamics based on the static game discussed in Section 3.1. We use generalized Smith-dynamics for channel selection and G-function based dynamics for rates. Combining them, we set up a framework of hybrid dynamics for the overall system.

The action of each user has two components $(\alpha_i, \mathbf{p}_i) \in \mathbb{R}_+ \times [0, 1]^J$. We use \mathbf{p}_i as strategies that determine the fitness of user i 's rate α_i to receiver j . The rate selection evolves according to the channel selection strategy \mathbf{P} . We may view channel selection as an inner game that involves a process on a short time scale but the rate selection is an outer game that represents the dynamical link via fitness on a longer time scale, [7], [8].

3.3.1 Learning the Weight Placed on Receiver

Let α be a fixed rate on the capacity region. We assume that user i occasionally experiments the weights p_{ij} with alternative receivers, keeping the new strategy if and only if it leads to a strict increase in payoff. If the choice of receivers' weights of some users decreases the payoff or violates the constraints due to a strategy change by another user, he starts a random search for a new strategy, eventually settling on one with a probability that increases monotonically with its

realized payoff. For the above generating function based dynamics, the weight of switching from receiver j to receiver j' is given by

$$\eta_{jj'}^i(\alpha, \mathbf{P}) = \max(0, u_{ij'}(\alpha, \mathbf{P}) - u_{ij}(\alpha, \mathbf{P}))^\theta, \quad \theta \geq 1$$

if the payoff obtained at receiver j' is greater the payoff obtained receiver j and the constraints are satisfied; otherwise, $\eta_{jj'}^i(p, \alpha) = 0$. The frequencies of uses of each receiver is then seen as the selection strategy of receivers.

The expected change at each receiver is the difference between the incoming flow and the outgoing flow. The dynamics is also called *generalized Smith dynamics* [2] and is given by

$$\dot{p}_{ij}(t) = \sum_{j' \in \mathcal{J}} p_{ij'}(t) \eta_{j'j}^i(\alpha, \mathbf{P}(t)) - p_{ij}(t) \sum_{j' \in \mathcal{J}} \eta_{jj'}^i(\alpha, \mathbf{P}(t)). \quad (19)$$

Let $\chi_{ij}(\alpha, \mathbf{P}(t)) := \sum_{j' \in \mathcal{J}} p_{ij'}(t) \eta_{j'j}^i(\alpha, \mathbf{P}(t)) - p_{ij}(t) \sum_{j' \in \mathcal{J}} \eta_{jj'}^i(\alpha, \mathbf{P}(t))$. Hence, the dynamics can be rewritten as $\dot{p}_{ij} = \chi_{ij}(\alpha, \mathbf{P}(t))$. For $\theta = 1$ the dynamics is known as *Smith dynamics* and has been used for describing the evolution of road traffic congestion in which the fitness is determined by the strategies chosen by all drivers. It has also been studied in the context of the resource selection in hybrid systems and migration constraint problem in wireless networks in [2].

Proposition 7. *Any equilibrium of the game Ξ with predetermined rates is a rest points of the generalized Smith dynamics (19).*

Proof. The transition rate between receivers preserves the sign in the sense that, for every user, incoming flow from the receiver j' to j is positive if and only if the constraints are satisfied and the payoff to j exceeds the payoff to j' . Let α be a feasible point. We first remark that if the right hand side of (19) is non-zero for some splitting strategy \mathbf{P} , then

$$\begin{aligned} d &:= \sum_{j \in \mathcal{J}} \dot{p}_{ij} u_{ij}(\alpha, \mathbf{P}) = \sum_{j \in \mathcal{J}} \chi_{ij} u_{ij}(\alpha, \mathbf{P}) \\ &= \sum_{j, j' \in \mathcal{J}} p_{ij'} (u_{ij}(\alpha, \mathbf{P}) - u_{ij'}(\alpha, \mathbf{P})) \eta_{j'j}^i \\ &= \sum_{j, j' \in \mathcal{J}} p_{ij'} \max[0, (u_{ij}(\alpha, \mathbf{P}) - u_{ij'}(\alpha, \mathbf{P}))] \eta_{j'j}^i \end{aligned}$$

which is strictly positive. Thus, if (α, \mathbf{P}) is a Nash equilibrium then (α, \mathbf{P}) satisfy the constraints, and $p_{ij} = 0$, or $\eta_{jj'}^i(\alpha, \mathbf{P}) = 0$. This implies that (α, \mathbf{P}) satisfies also $\chi(\alpha, \mathbf{P}) = 0$. \square

The following proposition says that the equilibria are exactly the rest point of (19).

Proposition 8. *Any rest point of the dynamics (19) is a Nash equilibrium of the game Ξ .*

The proof of Proposition 8 can be obtained by using Theorem III in [2]. Since the probability to switch from receiver j to j' is proportional to $\eta_{jj'}^i$, which preserves the sign of payoff difference, we can use the Theorems III in [2]. It follows that the dynamics generated by η satisfy the Nash stationarity property.

3.3.2 G-function Based Dynamics

We introduce here the generating fitness function (*G-function*) based dynamics with projection onto the capacity region. The G-function approach has been successfully applied to non-linear

continuous games by Vincent and Brown [7], [8]. It is appropriate for our hybrid model because we can regard the channel selection as the variables in a fitness function. Users choose channel selection probabilities to aim at increasing their fitness of their rate choice. In our rate allocation game, to deal with constraints, we use projection into capacity region in order to preserve the trajectories feasible. Starting from a point in the polytope \mathcal{C} , each user revises and updates its strategy according to a rate proportional to the gradient and its payoff subject to the capacity constraints. Let G_{ij} be the fitness generating function of user i at receiver j defined on $\mathbb{R}^N \times \mathbb{R}^{N \times J}$ satisfying

$$G_{ij}(v, \alpha, \mathbf{P}) \Big|_{v=\mathbf{P}_i} = \left(C_{j,\mathcal{N}} - p_{ij}\beta_{ij}(t) - \sum_{i' \in \mathcal{N} \setminus \{i\}} p_{i'j}\beta_{i'j}(t) \right),$$

if (α, \mathbf{P}) satisfies in the hybrid capacity region. Notice that the term $C_{j,\mathcal{N}} - \sum_{i' \neq i} p_{i'j}\beta_{i'j}(t)$ is maximum rate of i using channel j at time t . Hence, the G-function based dynamics is given by

$$\dot{\beta}_{ij} = -\bar{\mu} \left[p_{ij}\beta_{ij} - C_{j,\mathcal{N}} + \sum_{i' \neq i} p_{i'j}\beta_{i'j} \right] p_{ij}\beta_{ij}. \quad (20)$$

with initial conditions $\beta_{ij}(0) \leq C_{j,\{i\}}$, where $\beta = [\beta_{ij}]$ is defined in Proposition 5, which is of the same dimension as α , and $\alpha_i(t) = \sum_{j \in \mathcal{J}} \beta_{ij}(t)$; $\bar{\mu}$ is an appropriate parameter chosen for the rate of convergence.

3.3.3 Hybrid Dynamics

We now combine the two evolutionary game dynamics described in the previous subsections. Variables α and \mathbf{P} are both evolving in time. The overall dynamics are given by

$$\begin{cases} \dot{p}_{ij}(t) &= \sum_{j' \in \mathcal{J}} p_{ij'}(t) \eta_{jj'}^i(\alpha(t), \mathbf{P}(t)) - p_{ij}(t) \sum_{j' \in \mathcal{J}} \eta_{jj'}^i(\alpha(t), \mathbf{P}(t)) \\ \dot{\beta}_{ij}(t) &= -\bar{\mu} \left[p_{ij}(t)\beta_{ij}(t) - C_{j,\mathcal{N}} + \sum_{i' \neq i} p_{i'j}(t)\beta_{i'j}(t) \right] p_{ij}(t)\beta_{ij}(t) \\ \alpha_i(t) &= \sum_{j \in \mathcal{J}} \beta_{ij}(t), \beta_{ij}(0) \leq C_{j,\{i\}}, \forall j \in \mathcal{J}, i \in \mathcal{N} \end{cases} \quad (21)$$

All the equilibria of the hybrid evolutionary rate control and channel selection are rest point of the above hybrid dynamics. The following result can be obtained directly from Proposition 7 and (20).

Proposition 9. *Let (β^*, \mathbf{P}^*) be an interior rest points of the hybrid dynamics, i.e., $\beta_{ij}^* > 0$, $p_{ij}^* > 0$ and $\chi(\alpha^*, \mathbf{P}) = 0$. Then for all j ,*

$$\sum_{i=1}^N p_{ij}^* \beta_{ij}^* = C_{j,\mathcal{N}}; \quad \chi \left(\sum_{j=1}^N \beta_{ij}^*, \mathbf{P}^* \right) = 0.$$

3.4 Numerical Examples

In this subsection, we illustrate the evolutionary dynamics in (20) and (21) by examining a two-user and three-receiver communication system as depicted in Figure 4. Let $h_{i1} = 0.1$, $h_{i2} = 0.2$, $h_{i3} = 0.3$, for $i = \{1, 2\}$. Each transmission power P_i is set to 1 mW for all $i = 1, 2$ and the noise level is set to $\sigma^2 = -20$ dBm.

In the first experiment, we assume that the rates of the users are predetermined to be $\alpha = [10, 20]^T$, the Smith dynamics in (20) yield in Figure (5) and (6) the response of \mathbf{p}_1 and \mathbf{p}_2 . It can be seen that the dynamics converge very fast within less than half a second.

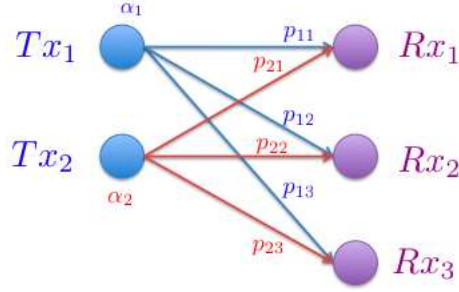


Figure 4: Two users and three receivers

In the second experiment, we assume that the probability matrix \mathbf{P} has been optimally found by the users using (19). Figures 7 and 8 show that the β values converge to an equilibrium from which we can find the optimal value for α . Since these dynamics are much slower compared to Smith dynamics on \mathbf{P} , our assumption of knowledge of optimal \mathbf{P} for a slowly varying α becomes valid.

In the next experiment, we simulate the hybrid dynamics in (21). Let the probability p_{ij} of user i choosing transmitter j and the transmission rates be initialized as follows:

$$\mathbf{P}(0) = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}, \quad \alpha(0) = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.$$

We let the parameter $\bar{\mu} = 0.9$. Figure 9 shows the evolution of the weights of user 1 on each of the receivers. The weights converge to be $p_{1j} = 1/3$ for all j within two seconds, leading to an unbiased choice among receivers. In Figure 10, we show the evolution of the weights of the second user on each receiver. At the equilibrium, $\mathbf{p}_2 = [0.3484, 0.4847, 0.1669]^T$. It appears that user 2 favors the second transmitter over the other ones. Since the utility u_{ij} is of the same form, the optimal response set K_i^* is naturally nonempty and contains all the receivers. Shown in Proposition 6, the probability of choosing a receiver at the equilibrium is randomized among the three receivers and can be determined by the rates α chosen by the users.

The β -dynamics determines the evolution of α in (21). In Figure 11, we see that the evolutionary dynamics yield $\alpha = [15.87, 23.19]^T$ at the equilibrium. It is easy to verify that they satisfy the capacity constraints outlined in Section 2. It converges within 5 seconds and appears to be much slower than in Figures 9 and 10. Hence, it can be seen that \mathbf{P} -dynamics may be seen as the inner loop dynamics while β -dynamics can be seen as an outer loop evolutionary dynamics. They evolve on two different time scales. In addition, thanks to Proposition 8, finding the rest points for the above dynamics ensures us finding the equilibrium.

4 Concluding Remarks

In this paper, we have studied an evolutionary multiple access channel game with a continuum action space and coupled rate constraints. We showed that the game has a continuum of strong equilibria which are 100% efficient in the rate optimization problem. We proposed the constrained Brown-von Neumann-Nash dynamics, Smith dynamics, and the replicator dynamics to study the stability of equilibria in the long run. In addition, we have introduced a hybrid multiple access game model and its corresponding evolutionary game-theoretic framework. We have analyzed the

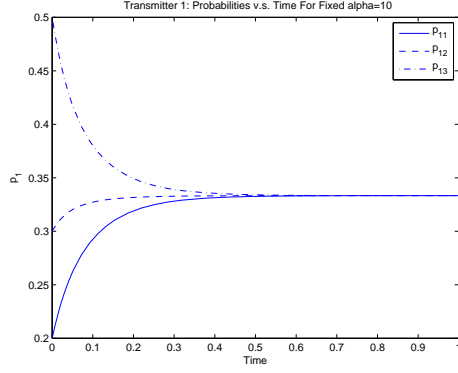


Figure 5: Transmitter 1: Probabilities v.s. Time For Fixed $\alpha_1=10$

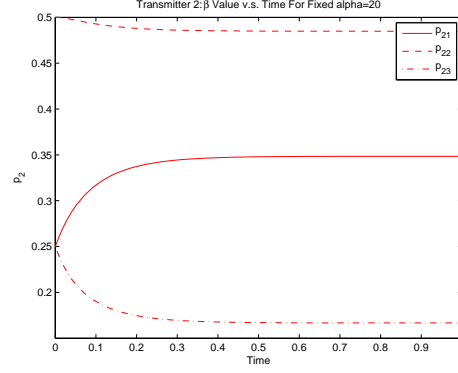


Figure 6: Transmitter 2: Probabilities v.s. Time For Fixed $\alpha_2=20$

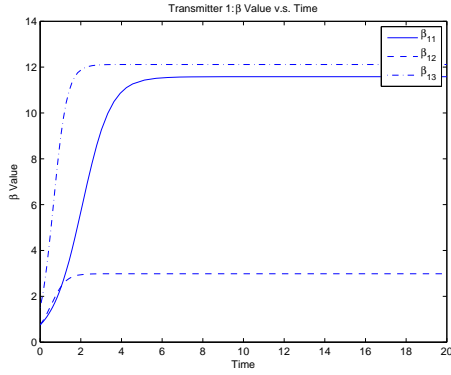


Figure 7: Transmitter 1: β Value v.s. Time

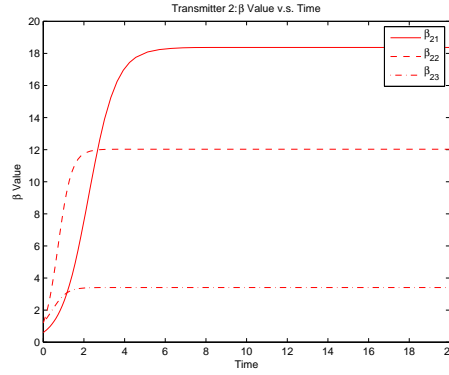


Figure 8: Transmitter 2: β Value v.s. Time

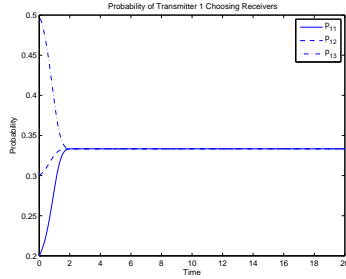


Figure 9: Probability of Transmitter 1 Choosing Receivers

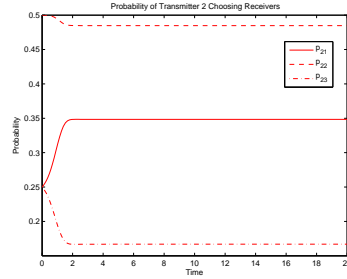


Figure 10: Probability of Transmitter 2 Choosing Receivers

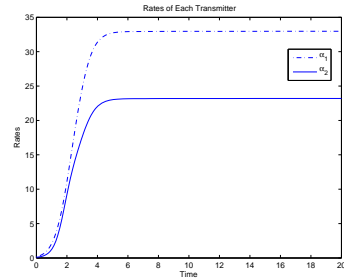


Figure 11: Rates of Each Transmitter

Nash equilibrium for the static game and suggested a system of evolutionary game dynamics based method to find it. It is found that the Smith dynamics for channel selections are a lot faster than the β -dynamics, and the combined dynamics yield a rest point that corresponds to the Nash equilibrium. An interesting extension that we leave for future research is to introduce a dynamic channel characteristics: the gains $h_{ij}(t)$ are *time-dependent random variables*. Another interesting

question is to find equilibria structure in the case of multiple access games with *non-convex capacity regions*.

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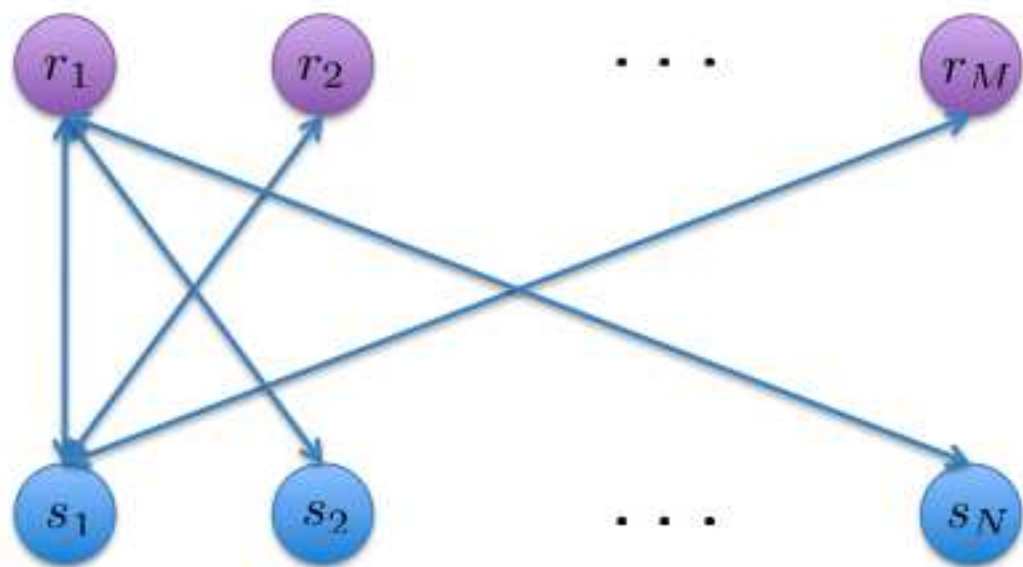
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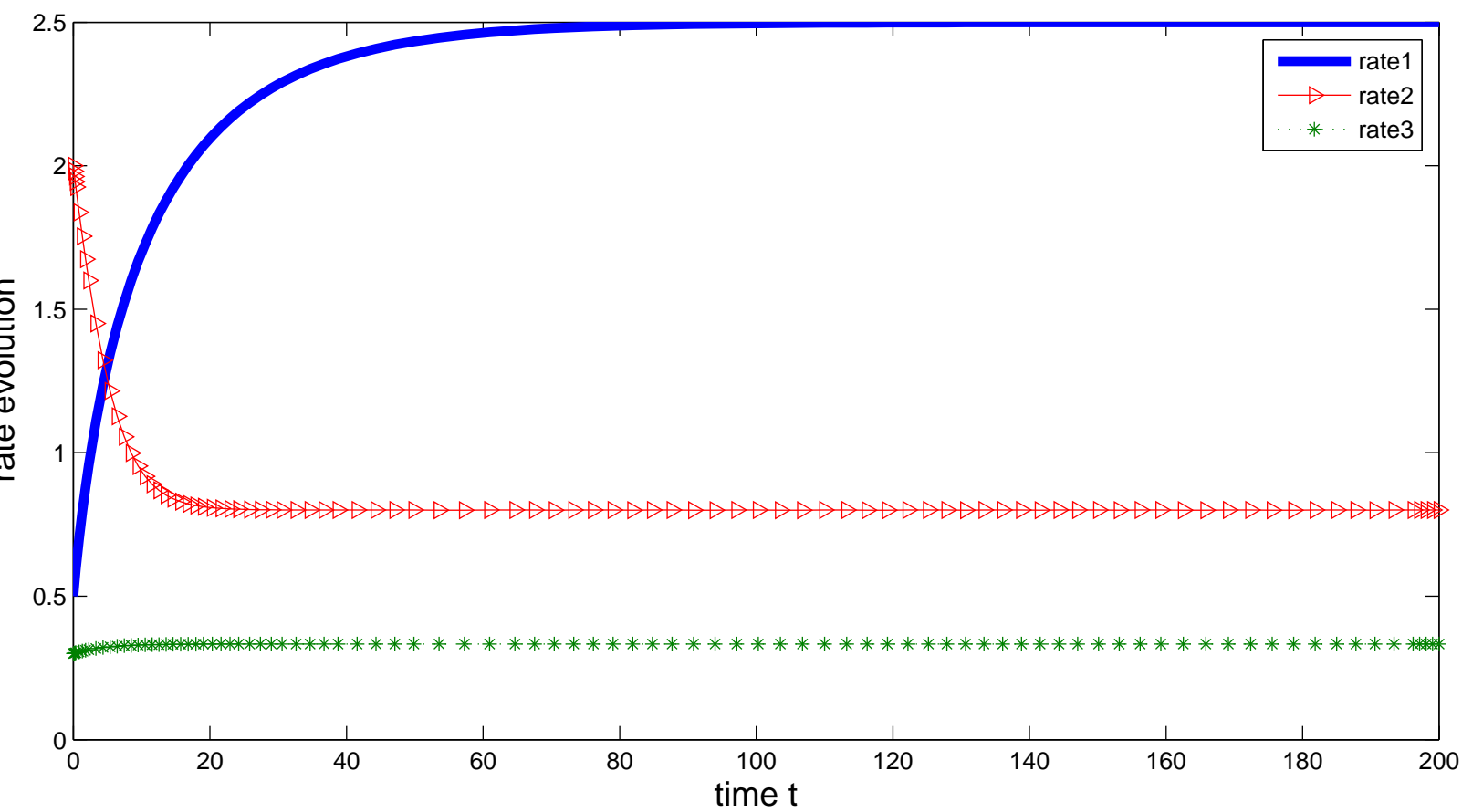
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G-function based dynamics

